Correlation Funktions of a System with Different Temperatures of the Particle Components

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We consider a plasma consisting of particle components with different temperatures. The components are uniformly distributed in the configuration space and Maxwellian in the velocity space. Pair correlations are assumed to be small and higher order correlations negligible. It is shown from the BBGKY-hierarchy that the influence of the electrons on the ion kinetics can be taken into account by treating the ions as dressed particles. The hierarchy for these dressed particles provides the ion-ion correlation function. The electron-ion pair correlation is calculated from the Poisson equation using the ion-ion correlation and relating the electron-ion pair distribution to the average potential. By the same procedure we derive the electron-electron correlation making use of the electron-ion correlation. The results are compared with those of other authors.

The knowledge of the reduced distribution functions provides complete information about the statistical behavior of a plasma. The evaluation of these functions is therefore one of the basic problems in plasma physics. The usual way to tackle this rather complicated problem is the solution of the BBGKY hierarchy derived from the Liouville equation. Since this hierarchy contains as many coupled partial differential equations as there are particles in the system, an exact solution is practically impossible. Therefore most investigations, even in the equilibrium case, truncate the hierarchy by neglecting higher than second order correlations.

We study here the special case of a stationary system in which each particle component is in equilibrium at a different temperature. The assumption of quasi-equilibrium for the single component is a good approximation since the energy exchange between the components is much smaller than the exchange within each component. Different temperatures for the electrons and ions are observed in many experiments.

Our derivation of the correlation functions was developed in connection with the problem of the lowering of the ionization energy for a plasma with different temperatures of the particle components [1, 2]. In investigations of the effective field in a plasma and the scattering of radiation by density fluctuations, other authors too studied the correlation functions [3–6]. Buneman [4] in his treatment of the scattered radiation from density fluctuations implies the problem of the pair correlation. The correlations calculated by Renau [5] using a more physically intuitive approach and by Kadomtsev [3] applying kinetic methods, do not agree with those of Salpeter [6]. Salpeter gives a derivation employing the collision free Boltzmann-equation and therewith an approximation which is equivalent to Debye's first treatment of the equilibrium case.

The following investigation derives rigorously the distribution functions from the BBGKY hierarchy within the frame of the pair approximation which neglects third and higher order correlations.

Concept

Subject of this investigation is a classical quasineutral system consisting of electrons and ions [6a]. N_k gives the total number of individuals of the kind k. T_e designates the electron temperature, T_i the temperature of the ions and neutrals. The plasma is uniformly distributed within a large volume V.

Following the general use we assume that third and higher order correlations are negligible.

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[6a] Neutrals may be present but do not affect our calculation.

Starting from the BBGKY hierarchy it is shown that the total interaction of one ion averaged over all electron microstates can be represented as the sum over effective pair interactions with the other ions. This means that the influence of the electrons on the ion kinetics can be taken into account by the use of "dressed ions".

We calculate the ion-ion correlation from the hierarchy for these dressed ions.

The electron-ion correlation can be expressed by the average potential in the environment of an ion. With this relation and the ion-ion correlation the average potential in the environment of an ion follows from POISSON's equation.

The electron-electron correlation is calculated by the same method using the average potential in the environment of an electron and the electron-ion correlation.

"Dressed Ions"

If $\mathcal{P}_{2N}$ with $2N = \sum_k N_k$ is the specific distribution function of the order $2N$, the corresponding reduced distribution function of the order s is defined by

$$\mathcal{P}_s = V^s \int \mathcal{P}_{2N} \, \mathrm{d}\boldsymbol{x}_{s+1} \dots \mathrm{d}\boldsymbol{x}_{2N}; \quad \mathrm{d}\boldsymbol{x}_i = \mathrm{d}\boldsymbol{r}_i \cdot \mathrm{d}\boldsymbol{v}_i \, . \tag{1}$$

In a multicomponent system $\mathcal{P}_s$ of course depends on the composition of the described s-configuration. However to simplify the notation we refrain from indicating this dependence. The general distribution function is given by

$$\mathcal{P}^{(s)} = \mathcal{P}_s \prod_k \frac{N_k!}{(N_k - \nu_k)!} \, . \tag{2}$$

The index k denotes the different particle components. ν_k is the number of particles of the component k in the considered s-configuration.

From the LIOUVILLE equation one obtains for $\mathcal{P}_s$ the hierarchy

$$\frac{\partial}{\partial t} \mathcal{P}_s + \sum_{i=1}^{s} \boldsymbol{v}_i \frac{\partial}{\partial \boldsymbol{r}_i} \mathcal{P}_s - \sum_{i,j=1}^{s}{}' \frac{1}{m_i} \frac{\partial}{\partial \boldsymbol{v}_i} \mathcal{P}_s \frac{\partial}{\partial \boldsymbol{r}_i} \varphi_{ij} - \sum_k \frac{N_k - \nu_k}{V} \sum_{i=1}^{s} \frac{1}{m_i} \int \frac{\partial}{\partial \boldsymbol{v}_i} {}^k\mathcal{P}_{s+1} \frac{\partial}{\partial \boldsymbol{r}_i} \varphi_{i,\, s+1} \, \mathrm{d}\boldsymbol{x}_{s+1} = 0 \tag{3}$$

where φ_{ij} is the COULOMB pair energy. The indices i, j denote particles of the s-configuration. ${}^k\mathcal{P}_{s+1}$ describes a configuration which contains one particle of the component k in addition to the particles of the original s-configuration. m is the particle mass. We now treat the s-configuration including all ions [7].

Since we have MAXWELLian distributions F_i in the velocity space the distribution functions $\mathcal{P}_s$ represent themselves as

$$\mathcal{P}_s = P_s \prod_{i=1}^{s} F_i \tag{4}$$

where P_s are the specified distribution functions in the configuration space. If $\langle \Phi_i \rangle^{(N)}$ designates the average potential energy of the particle (i) in the N-configuration of all ions

$$\langle \Phi_i \rangle^{(N)} = \int \sum_{\nu=1}^{2N}{}' \varphi_{i\nu} P_{2N} \, \mathrm{d}\boldsymbol{r}_{N+1} \dots \mathrm{d}\boldsymbol{r}_{2N} \Big/ \int P_{2N} \, \mathrm{d}\boldsymbol{r}_{N+1} \dots \mathrm{d}\boldsymbol{r}_{2N} \tag{5}$$

we have the relation

$$\frac{\partial}{\partial \boldsymbol{r}_i} \langle \Phi_i \rangle^{(N)} = \sum_{j=1}^{N}{}' \frac{\partial}{\delta \boldsymbol{r}_i} \varphi_{ij} + \frac{1}{V} \int \frac{P(N+1)}{P(N)} \frac{\partial}{\partial \boldsymbol{r}_i} \varphi_{i,\, N+1} \, \mathrm{d}\boldsymbol{r}_{N+1} + \frac{1}{V} \int \varphi_{i,\, N+1} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \left(\frac{P(N+1)}{P(N)} \right) \mathrm{d}\boldsymbol{r}_{N+1} \, . \tag{6}$$

We now show that the last term on the right hand side of eq. (6) disappears within the frame of our investigation.

Neglecting all higher order correlations we have

$$\begin{aligned} P_N &= \prod_j P_1(\boldsymbol{r}_j) \prod_{(ij)} \{ 1 + g_{ij}^*(\boldsymbol{r}_i, \boldsymbol{r}_j) \} \\ &= \prod_{(ij)} P_2^*(\boldsymbol{r}_i, \boldsymbol{r}_j) \end{aligned} \tag{7}$$

for the specific distribution function [8]. The last term of this equality uses the fact that $P_1 = 1$ in our system. The function g_{ij} is the pair correlation function. The symbol $(i\,j)$ means all combinations of the configuration particles. After simple transformations

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[7] For the sake of simplicity we consider from now on only one kind of ions ($N_\mathrm{i} = N_\mathrm{e} = N$). The generalization to many ion components is straight forward.

[8] The star distinguishes the pair distribution in a system with a specified configuration of all ions from the usual one (P_2) without any specification except for the two particles under consideration.

we obtain for the general distribution functions the relation

$$\frac{P(N+1)}{P(N)} = N \cdot \prod_j P_2^*(\boldsymbol{r}_j, \boldsymbol{r}_{N+1}). \tag{8}$$

Therefore the last term on the right hand side of eq. (6) reads

$$\frac{1}{V} \varphi_{i,N+1} \int \frac{\partial}{\partial \boldsymbol{r}_i} \left(\frac{P(N+1)}{P(N)} \right) \mathrm{d}\boldsymbol{r}_{N+1} = n \int \varphi_{i,N+1} \frac{\partial}{\partial \boldsymbol{r}_i} g_{i,N+1} \, \mathrm{d}\boldsymbol{r}_{N+1}. \tag{9}$$

Since in our system the functions g_{ij}^* and φ_{ij} depend only on the distance $|\boldsymbol{r}_i - \boldsymbol{r}_j|$ we have

$$\int \varphi_{i,N+1} \frac{\partial}{\partial \boldsymbol{r}_i} \frac{P(N+1)}{P(N)} \mathrm{d}\boldsymbol{r}_{N+1} = 0 \tag{10}$$

for symmetry reasons.

Inserting eqs. (8) and (10) into (6) we find

$$\frac{\partial}{\partial \boldsymbol{r}_i} \langle \Phi_i \rangle^{(N)} = \sum_j{}' \frac{\partial}{\partial \boldsymbol{r}_i} \varphi_{ij} + n \int \prod_j P_2^*(\boldsymbol{r}_j, \boldsymbol{r}_{N+1}) \frac{\partial}{\partial \boldsymbol{r}_i} \varphi_{i,N+1} \, \mathrm{d}\boldsymbol{r}_{N+1} \tag{11}$$

For the electron-ion pair distribution function $P_2(\boldsymbol{r}_\mathrm{i}, \boldsymbol{r}_\mathrm{e})$ the hierarchy provides

$$\boldsymbol{v}_\mathrm{i} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} P_2 + \boldsymbol{v}_\mathrm{e} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} P_2 + \frac{\boldsymbol{v}_\mathrm{i}}{k\,T_\mathrm{i}} P_2 \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_\mathrm{ie} + \frac{\boldsymbol{v}_\mathrm{e}}{k\,T_\mathrm{e}} P_2 \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \varphi_\mathrm{ie} + \frac{\boldsymbol{v}_\mathrm{i} n}{k\,T_\mathrm{i}} \sum_k \int {}^kP_3 \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_{\mathrm{i},3} \, \mathrm{d}\boldsymbol{r}_3 + \frac{\boldsymbol{v}_\mathrm{e} n}{k\,T_\mathrm{e}} \sum_k \int {}^kP_3 \, \varphi_{\mathrm{e},3} \, \mathrm{d}\boldsymbol{r}_3 = 0 \tag{12}$$

where the index (e) designates the electron.

Equ. (12) down to equ. (18) hold identically for the functions P^*, $\langle \Phi \rangle^*$ except that the sums do not include the ion component. $\langle \Phi \rangle^*$ denotes the average potential pair energy in the N-configuration.

Under the conditions

$$v_\mathrm{i}/(k\,T_\mathrm{i}) \ll v_\mathrm{e}/(k\,T_\mathrm{e})\,; \quad v_\mathrm{i} \ll v_\mathrm{e} \tag{13}$$

equation (12) reduces to

$$k\,T_\mathrm{e} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \ln P_2 = -\frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \varphi_\mathrm{ie} - \frac{n}{P_2} \sum_k \int {}^kP_3 \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \varphi_\mathrm{e3} \, \mathrm{d}\boldsymbol{r}_3. \tag{14}$$

For the potential energy of the electron – ion pair we obtain

$$\frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \langle \Phi_\mathrm{e} \rangle^{(2)} = \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \varphi_\mathrm{ie} + \frac{n}{P_2} \sum_k \int {}^kP_3 \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \varphi_\mathrm{e3} \, \mathrm{d}\boldsymbol{r}_3. \tag{15}$$

Comparison of eqs. (14) and (15 leads to the relation

$$k\,T_\mathrm{e} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \ln P_2 = -\frac{\partial}{\partial \boldsymbol{r}_\mathrm{e}} \langle \Phi_\mathrm{e} \rangle^{(2)} \tag{16}$$

and since P_2 depends only on the distance of the two particles

$$P_2 = A \cdot \exp\left\{ -\frac{1}{k\,T_\mathrm{e}} \langle \Phi_\mathrm{e} \rangle^{(2)} \right\}. \tag{17}$$

The proportionality constant A follows from normalization to be

$$A = V^2 \Big/ \int \exp\left\{ -\frac{1}{k\,T_\mathrm{e}} \langle \Phi_\mathrm{e} \rangle^{(2)} \right\} \mathrm{d}\boldsymbol{r}_\mathrm{i} \, \mathrm{d}\boldsymbol{r}_\mathrm{e}. \tag{18}$$

Introducing eq. (17) into eq. (11) leads to

$$\frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \langle \Phi_\mathrm{i} \rangle^{(N)} = \sum_j{}' \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_\mathrm{ij} + A\,n \int \exp\left\{ -\frac{1}{k\,T_\mathrm{e}} \sum_j \langle \Phi(\boldsymbol{r}_j, \boldsymbol{r}_{N+1}) \rangle^{*(2)} \right\} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_{\mathrm{i},N+1} \, \mathrm{d}\boldsymbol{r}_{N+1}. \tag{19}$$

We represent the potential energy $\langle \Phi_\mathrm{i} \rangle^{(N)}$ by the Ansatz

$$\langle \Phi_\mathrm{i} \rangle^{(N)} = \sum_j{}' \Phi_\mathrm{ij} = \sum{}' \langle \Phi_\mathrm{ij} \rangle^{*(2)} \tag{20}$$

and linearize eq. (19) to obtain

$$\sum_j{}' \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \Phi_\mathrm{ij} = \sum_j{}' \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_{\mathrm{i}j} + A\,n \int \left(1 - \frac{1}{k\,T_\mathrm{e}} \sum_j \Phi_{j,N+1} \right) \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_{\mathrm{i},N+1} \, \mathrm{d}\boldsymbol{r}_{N+1}. \tag{21}$$

The linearization is consistent within the frame of our assumption $g_{ij} \ll 1$.

Applying symmetry considerations and the normalization

$$\int \exp\left\{ -\frac{1}{k\,T_\mathrm{e}} \langle \Phi \rangle^{(2)} \right\} \mathrm{d}\boldsymbol{r}_1 \, \mathrm{d}\boldsymbol{r}_2 = V^2 \tag{22}$$

eq. (21) takes the form

$$\sum_j{}' \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \Phi_\mathrm{ij} = \sum_j{}' \varphi_\mathrm{ij} - \frac{n}{k\,T_\mathrm{e}} \sum_j \int \Phi_{j,N+1} \frac{\partial}{\partial \boldsymbol{r}_\mathrm{i}} \varphi_{\mathrm{i},N+1} \, \mathrm{d}\boldsymbol{r}_{N+1}. \tag{23}$$

Since
$$\int \Phi_{\mathrm{i},N+1} \frac{\partial}{\partial \boldsymbol{r}_{\mathrm{i}}} \varphi_{\mathrm{i},N+1} \, \mathrm{d}\boldsymbol{r}_{N+1} = 0 \tag{24}$$
holds we finally arrive with the use of the relation
$$\Phi_{\mathrm{i}j}(\boldsymbol{r}-\boldsymbol{r}') = -\Phi_{\mathrm{i},N+1}(\boldsymbol{r}-\boldsymbol{r}') \tag{25}$$
at
$$\frac{\partial}{\partial \boldsymbol{r}} \Phi(r) = \frac{\partial}{\partial \boldsymbol{r}} \varphi(r) - \frac{n}{k T_{\mathrm{e}}} \int \Phi(r') \frac{\partial}{\partial \boldsymbol{r}} \varphi(|\boldsymbol{r}-\boldsymbol{r}'|) \, \mathrm{d}r' \tag{26}$$

Since this equation is identical with the equation for $\langle \Phi_{\mathrm{i}j} \rangle^{*(2)}$ derived from (15) using (8) and (17) the Ansatz (20) is justified.

The well known solution of the equation (26) which is finite for large distances reads
$$\Phi(r) = e^2/r \cdot \exp(-\varkappa_{\mathrm{e}} r) \tag{27}$$
with the screening constant
$$\varkappa_{\mathrm{e}}^2 = 4\pi n e^2/(k T_{\mathrm{e}}) \, . \tag{28}$$

Therewith we have shown that the average potential energy of a configuration of all ions can be represented as a sum over effective ion-pair energies. That means the influence of the averaging process over all electron configurations on the ion interaction can be taken into account by the use of dressed ions.

The ion-ion correlation

The ion-ion correlation is now calculated from the BBGKY hierarchy for the dressed ions. This hierarchy is obtained from eq. (3) substituting the Coulomb potential by the effective potential (27) and omitting in the last term the summation over the electron component. Neglecting consistently the higher order correlations we find for the pair correlation function the equation
$$\frac{\partial}{\partial \boldsymbol{r}} g(r) = \frac{\partial}{\partial \boldsymbol{r}} \Phi(r) \frac{1}{k T_{\mathrm{i}}} - \frac{\varkappa^2_{\mathrm{i}}}{4\pi e^2} \int g(r') \frac{\partial}{\partial \boldsymbol{r}} \Phi(|\boldsymbol{r}-\boldsymbol{r}'|) \, \mathrm{d}r' \tag{29}$$
with
$$\varkappa_{\mathrm{i}}^2 = 4\pi n e^2/(k T_{\mathrm{i}}) \, . \tag{30}$$

Fourier transformation of this equation produces
$$P_2(\mathrm{i},\mathrm{i}) = 1 - \frac{e^2}{k T_{\mathrm{i}} r} \exp(-\varkappa r) \tag{31}$$
with the screening constant
$$\varkappa^2 = \varkappa_{\mathrm{i}}^2 + \varkappa_{\mathrm{e}}^2. \tag{32}$$

The electron-ion correlation

To calculate the electron-ion pair distribution we apply the Poisson equation to the environment of an ion using the procedure outlined in the concept. This requires the knowledge of the electron and ion density in the neighbourhood of an ion. The ion density is given by the ion pair distribution according to eq. (31). For the electron density eq. (17) provides after linearization
$$P_2(\mathrm{e},\mathrm{i}) = 1 - \frac{\langle \Phi_{\mathrm{e}} \rangle^{(2)}}{k T_{\mathrm{e}}} = 1 + \frac{e \, \Psi_{\mathrm{i}}(r)}{k T_{\mathrm{e}}} \, . \tag{33}$$

Therefore the differential equation for the average potential $\Psi_{\mathrm{i}}(r)$ in the environment of an ion reads
$$\Delta \Psi_{\mathrm{i}}(r) = 4\pi e n \left(1 + \frac{e \, \Psi_{\mathrm{i}}(r)}{k T_{\mathrm{e}}}\right) - 4\pi e n \left(1 - \frac{e^2}{k T_{\mathrm{i}}} \frac{1}{r} \exp\{-\varkappa r\}\right). \tag{34}$$

The solution of this equation taking into account the boundary conditions is
$$\Psi_{\mathrm{i}}(r) = \frac{e}{r} \cdot \exp\{-\varkappa r\} \, . \tag{35}$$

The electron-ion pair distribution reads consequently
$$P_2(\mathrm{e},\mathrm{i}) = 1 + \frac{e^2}{k T_{\mathrm{e}}} \frac{1}{r} \exp(-\varkappa r) \, . \tag{36}$$

The electron-electron correlation

The procedure to calculate the electron correlation is analog to the method used in the preceding chapter. We write down the Poisson equation in the neighbourhood of an electron. The ion density is given by the electron-ion pair distribution found in eq. (36). For the electron pair distribution we find from the BBGKY hierarchy in the same manner as described for $P_2(\mathrm{e},\mathrm{i})$ the relation
$$P_2(\mathrm{e},\mathrm{e}) = 1 + e \, \Psi_{\mathrm{e}}(r)/(k T_{\mathrm{e}}) \, . \tag{37}$$

The average potential in the environment of an electron is therefore governed by the differential equation
$$\Delta \Psi_{\mathrm{e}}(r) = 4\pi e n \left(1 + \frac{e \, \Psi_{\mathrm{e}}(r)}{k T_{\mathrm{e}}}\right) - 4\pi e n \left(1 + \frac{e^2}{k T_{\mathrm{e}}} \frac{1}{r} \exp\{-\varkappa r\}\right). \tag{38}$$

The solution of the equation accounting for the boundary conditions is
$$\Psi_{\mathrm{e}}(r) = -\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}} \frac{e}{r} \exp(-\varkappa r) - \left(1 - \frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}\right) \frac{e}{r} \exp(-\varkappa_{\mathrm{e}} r) \tag{39}$$

which determines the pair distribution function

$$P_2(\mathrm{e},\mathrm{e}) = 1 - \frac{T_\mathrm{i}}{k\,T_\mathrm{e}^2}\,\frac{e^2}{r}\exp(-\varkappa\,r) - \frac{1}{k\,T_\mathrm{e}}\left(1-\frac{T_\mathrm{i}}{T_\mathrm{e}}\right)\frac{e^2}{r}\exp(-\varkappa_\mathrm{e}\,r). \qquad (40)$$

Discussion

We compare our results with those of SALPETER and KADOMTSEV. With respect to the work of BUNEMAN and RENAU we refer to the discussion SALPTER's paper.

The results of KADOMTSEV do not agree with those derived here. He gives the electron-electron correlation

$$g_\mathrm{ee} = -\frac{e^2}{k\,T_\mathrm{e}\,r}\exp(-\varkappa\,r) \qquad (41)$$

in contrast to our correlation function (40). KADOMTSEV's kinetic treatment is in principle correct. The deviation of his results is due to an error in the evaluation of the following system of equations

$$\begin{aligned}
g_\mathrm{ei}(r) - \frac{n\,e^2}{k\,T_\mathrm{e}}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\{g_\mathrm{ii}(r')-g_\mathrm{ei}(r')\}\,\mathrm{d}\mathbf{r}' &= \frac{e^2}{k\,T_\mathrm{e}\,r},\\
g_\mathrm{ee}(r) - \frac{n\,e^2}{k\,T_\mathrm{e}}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\{g_\mathrm{ei}(r')-g_\mathrm{ee}(r')\}\,\mathrm{d}\mathbf{r}' &= -\frac{e^2}{k\,T_\mathrm{i}\,r},\\
g_\mathrm{ii}(r) - \frac{n\,e^2}{k\,T_\mathrm{i}}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\{g_\mathrm{ei}(r')-g_\mathrm{ii}(r')\}\,\mathrm{d}\mathbf{r}' &= -\frac{e^2}{k\,T_\mathrm{e}\,r}.
\end{aligned} \qquad (42)$$

The correlation (41) does not satisfy this system, whereas (40) is a solution. This can be shown simply by insertion.

The results derived by SALPETER using a procedure similar to DEBYE's original approach agree with those justified here by a general kinetic treatment. In addition we realize that application of these results must be restricted to the temperature range

$$m_\mathrm{i}/m_\mathrm{e} \gg T_\mathrm{e}/T_\mathrm{i} \gg m_\mathrm{e}/m_\mathrm{i} \qquad (43)$$

due to eq. (13).

Besetzungsdichten angeregter He I-Atome in einem nicht-thermischen Plasma

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For nonthermal plasmas, the population densities of the ground level and the lower lying excited states can deviate considerably from a SAHA- and BOLTZMANN population density which are only valid for plasmas in (local) thermal equilibrium. To obtain the actual population densities of the lower lying states of the He I-Atom in a nonthermal, optically thin as well as in a partially optically thick plasma the corresponding rate equations have been solved. Due to the metastable state 2^3S we have distinguished between the singulet and the triplet system. The coupling between the two systems has been accomplished by appropriate collisional processes. Numerical values are given for the population densities in terms of „SAHA"-population densities for both the lower singlet- and triplet levels in the case of an optically thin plasma. The relaxation times necessary for establishing a steady-state have been calculated. The influence of the triplet-system on the particle densities of the singlet-system is shown. For low electron densities the ratio $n_i(\text{triplet})/n_i(\text{singlet})$ becomes larger than 3. By solving the complete system of rate equations and comparing the solutions with measured densities one can derive unknown total cross sections for reactions between excited states.

Im Falle (lokalen) thermodynamischen Gleichgewichtes (L.T.E.) ist die Besetzungsdichte der angeregten Niveaus eines Atoms oder Ions entsprechend einer BOLTZMANN-Verteilung gegeben. Die BOLTZMANN-Verteilung ist jedoch nicht mehr sichergestellt bei Plasmen geringer Teilchendichten und kleinen geometrischen Abmessungen der zu untersuchenden Plasmaschichten, da die Anzahl der Stoßprozesse abnimmt und Absorption von Resonanzstrahlung klein wird. Der Grenzfall eines stationären